TYPICAL LAWS OF HEREDITY[1]

III.

If a graphic representation is desired, which will give the absolute number of survivors at each degree, we must shape the rampart which forms nature's target so as to be highest in the middle and to slope away at each side according to the law of deviation. Thus Fig. 6 represents the curved rampart before it has been aimed at; Fig. 7, afterwards.

I have taken a block of wood similar to Fig. 4, to represent the rampart; it is of equal height throughout. A cut has been made at right angles to its base with a fret-saw, to divide it in two portions—that which would remain after it had been breached, Fig. 5, and the cast of the breach. Then a second cut with the fret-saw has been made at right angles to its face, to cut out of the rampart an equivalent to the heap of pellets that represents the original population. The gap that would be made in the heap and the cast that would fill the gap are curved on two faces, as in the model. This is sufficiently represented in Fig. 7.

The operation of natural selection on a population already arranged according to the law of deviation is represented more completely in an apparatus, Fig. 8, which I will set to work immediately.

It is faced with a sheet of glass. The heap, as shown in the upper compartment of the apparatus, is three inches in thickness, and the pellets rest on slides. Directly below the slides, and running from side to side of the apparatus, is a curved partition, which will separate the pellets as they fall upon it, into two portions, one that runs to waste at the back, and another that falls to the front, and forms a new heap. The curve of the partition is a curve of deviation. The shape of this heap is identical with the cast of the gap in Fig. 7. It is highest and thickest in the middle, and it fines away towards either extremity. When the slide upon which it rests is removed, the pellets run down an inclined plane that directs them into a frame of uniform and shallow depth. The pellets from the deep central compartments (it has been impossible to represent in the diagram as many of these as there were in the apparatus) will stand very high from the bottom of the shallow frame, while those that came from the distant compartments will stand even lower than they did before. It follows that the selected pellets form, in the lower compartment, a heap of which the scale of deviation is much more contracted than that of the heap from which it was derived. It is perfectly normal in shape, owing to an interesting theoretical property of deviation (see formulæ at end of this memoir).

Productiveness follows the same general law as survival, being a percentage of possible production, though it is usual to look on it as a simple multiple, without dividing by the 100. In this case the front face of each compartment in the upper heap represents the number of the parents of the same class, and the depth of the partition below that compartment represents the average number that each individual of that class produces.

To sum up. We now see clearly the way in which the resemblance of a population is maintained. In the purely typical case, each of the processes of heredity and selection is subject to a well-defined and simple law, which I have formulated in the appendix. It follows that when we know the values of 1° in the several curves of family variability, productiveness, and survival, and when we know the co-efficient of reversion, we know absolutely all about the ways in which that characteristic will be distributed among the population at large.

I have confined myself in this explanation to purely typical cases, but it is easy to understand how the actions of the processes would be modified in those that were not typical. Reversion might not be directed towards the mean of the race, neither productiveness nor survival might be greatest in the medium classes, and none of their laws may be strictly of the typical character. However, in all cases the general principles would be the same. Again, the same actions that restrain variability would restrain the departure of average values beyond certain limits. The typical laws are those which most nearly express what takes place in nature generally; they may never be exactly correct in any one case, but at the same time they will always be approximately true and always serviceable for explanation. We estimate through their means the effects of the laws of sexual selection, of productiveness, and of survival, in aiding that of reversion in bridling the dispersive effect of family variability. They show us that natural selection does not act by carving out each new generation according to a definite pattern on a Procrustean bed, irrespective of waste. They also explain how small a contribution is made to future generations by those who deviate widely from the mean, either in excess or deficiency, and enable us to calculate whence the deficiency of exceptional types is supplied. We see by them that the ordinary genealogical course of a race consists in a constant outgrowth from its centre, a constant dying away at its margins, and a tendency of the scanty remnants of all exceptional stock to revert to that mediocrity, whence the majority of their ancestors originally sprang.

Appendix.

I will now proceed to formulate the typical laws. In what has been written, 1° of deviation has been taken equal to the "probable error" = C × 0·4769 in the well-known formula $y = \frac{1}{c\sqrt{\pi}} . e^{-\frac{x^2}{c^2}}$. According to this, if x = amount of deviation in feet, inches, or any other external unit of measurement, then the number of individuals in any sample who deviate between x and $x + \delta x$ will vary as $e^{-\frac{x^2}{c^2}} \delta x$ (it will be borne in mind that we are for the most part not concerned with the coefficient in the above formula).

Let the modulus of deviation (c) in the original population, after the process has been gone through, of converting the measurements of all its members (in respect to the characteristic in question), to the adult male standard, be written c_0.

1. Sexual selection has been taken as *nil*, therefore the population of "parentages" is a population of which each unit consists of the mean of a couple taken indiscriminately. This, as well known, will conform to the law of deviation, and its modulus which we will write c_1 has already been shown to be equal to $\frac{1}{\sqrt{2}} . c_0$.

2. Reversion is expressed by a fractional coefficient of the deviation, which we will write r. In the "reverted" parentages (a phrase whose meaning and object have already been explained)

$$y = \frac{1}{r c \sqrt{\pi}} . e^{-\frac{x^2}{r^2 c^2}}.$$

In short, the population, of which each unit is a reverted parentage, follows the law of deviation, and has its modulus, which we will write c_2, equal to $r c_1$.

3. Productiveness:—We saw that it followed the law of deviation; let its modulus be written f. Then the number of children to each parentage that differs by the amount of x from the mean of the parentages generally (*i.e.*, from the mean of the race), will vary as $e^{-\frac{x^2}{f^2}}$; but the number of such parentages varies as $e^{-\frac{x^2}{c_2{}^2}}$, therefore if each child

[1] Lecture delivered at the Royal Institution, Friday evening, February , by Francis Galton, F.R.S. Continued from p. 514.

absolutely resembled his parent, the number of children who deviated x would vary as $e^{-\frac{x^2}{f^2}} \times e^{-\frac{x^2}{c_2^2}}$, or as $e^{-x^2\left\{\frac{1}{f^2}+\frac{1}{c_2^2}\right\}}$. Hence the deviations of the children in their amount and frequency would conform to the law, and the modulus of the population of children in the supposed case of absolute resemblance to their parents, which we will write c_3, is such that—

$$\frac{1}{c_3} = \sqrt{\left(\frac{1}{f^2}+\frac{1}{c^2}\right)}.$$

We may, however, consider the parents to be multiplied

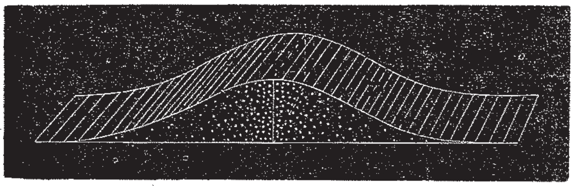

FIG. 6.

and the productivity of each of them to be uniform. It is more convenient than the converse supposition and it comes to the same thing. So we will suppose the reverted parentages to be more numerous but equally prolific, in which case their modulus will be c_3, as above.

4. Family variability was shown by experiment to follow the law of deviation, its modulus, which we will write v, being the same for all classes. Therefore the amount of deviation of any one of the offspring from the mean of his race is due to the combination of two influences, the deviation of his "reverted" parentage and his own family variability; both of which follow the law of deviation. This is obviously an instance of the well-known law of the "sum of two fallible measures" (Airy, "Theory of Errors," § 43). Therefore the modulus of the population in the present stage, which we will write c_4, is equal to $\sqrt{(v^2 + c_3^2)}$.

5. Natural selection follows, as has been explained, the same general law as productiveness. Let its modulus be written s; then the percentage of survivals among children, who deviate x from the mean, varies as $e^{-\frac{x^2}{s^2}}$, and for the same reasons as those already given, its effect will be to leave the population still in conformity

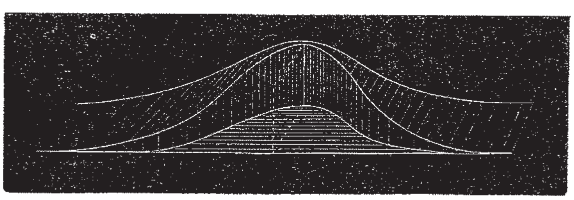

FIG. 7.

with the law of deviation, but with an altered modulus, which we will write c_5, and

$$\frac{1}{c_5} = \sqrt{\left(\frac{1}{s^2}+\frac{1}{c_4^2}\right)}.$$

Putting these together we have, starting with the original population having a modulus $= c_0$:—

1. $c_1 = \frac{1}{\sqrt{2}} c_0.$

2. $c_2 = rc_1.$

3. $c_3 = \sqrt{\left\{\frac{f^2 c_2^2}{f^2 + c_2^2}\right\}}.$

4. $c_4 = \sqrt{\left\{v^2 + c_3^2\right\}}.$

5. $c_5 = \sqrt{\left\{\frac{s^2 c_4^2}{s^2 + c_4^2}\right\}}.$

And lastly, as the condition of maintenance of statistical resemblance in consecutive generations:—

6. $c_5 = c_0.$

Hence, given the coefficient r and the moduli v, f, s, the value of c_0 (or c_5) can be easily calculated.

In the case of simple descent, which was the one first

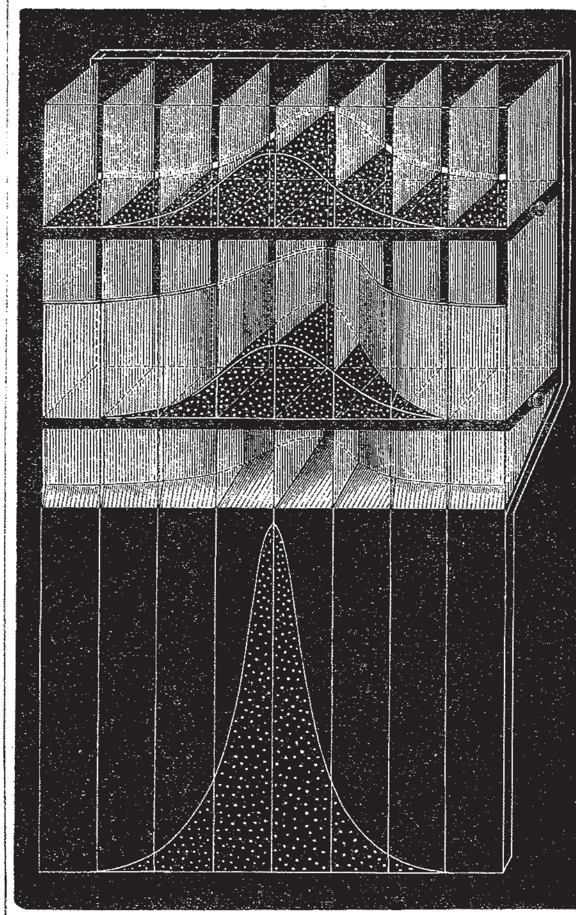

FIG. 8.

considered, we have nothing to do with c_0, but begin from c_1. Again, as both fertility and natural selection are in this case uniform, the values of f and s are infinite. Consequently our equations are reduced to—

$$c_2 = rc_1\,;\ c_4 = \sqrt{\left\{v^2 + c_2^2\right\}}\,;\ c_4 = c_1,$$

whence

$$c_1^2 = \frac{v^2}{1 - r^2}.$$

CARL FRIEDRICH GAUSS

BORN APRIL 30, 1777, DIED FEBRUARY 23, 1855.[1]

DE MORGAN in his "Budget of Paradoxes" (p. 187), tells the following story:—The late Francis Baily wrote a singular book, "Account of the Rev. John Flamsteed, the first Astronomer-Royal:" it was published by the Admiralty for distribution, and the author drew up the distribution list.

[1] We adopt the date given by the Baron Sartorius von Waltershausen in his "Gauss. Zum Gedächtniss," Leipzig, 1856. Encyclopedists and other authorities are pretty equally divided between this date and April 23. All the English Cyclopædias we have consulted, with the exception of Chambers's (1874), give April 23. We may also mention that on the list of students at the Collegium Carolinum the name is Johann Friedrich Karl Gauss. We have followed Gauss himself in our heading.